<u>9th Lecture</u>

Fatigue Construction Damage at Time-Variable Loading

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9.1– Introduction

Pulsing load of structures causes fatigue damage of material. Such damage is a consequence of plastic deformation located in the notch neighbourhood. It is not really a question of a notch in the literal sense (see the 6th Lecture) but of all the points where the material significantly changes its properties, deformation rigidity or shows discontinuities of various types.

The notches also influence material surface layers having reduced rigidity and usually micro-geometrically irregular shape. The construction notches, material defects and surface unevenness have large influence on the fatigue strength of bodies. Only a certain limited volume of material is subjected to the concentrated plastic strain during elastic cyclic deformation of the whole body. It shows that the fatigue damage is a local process and depends thus on local material properties (see Fig. 9.1). As a fatigue fracture occurs finally at the same time, it is obvious that the plastic deformation is preceded by crack root propagation; the whole evolution of the consistency defect has to be gradual and depends on the number of loading changes.

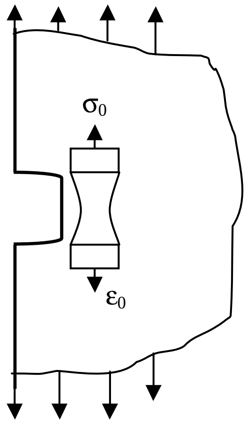

Fig.9.1

Local mechanical properties of material change due to the cyclic plastic deformations. The standards of these properties are not the results of static testing of the material, because they show just average properties throughout the material cross section, mostly at homogenous or linear distribution of stresses and they do not involve changes caused by mechanical aging of material due to manifold repeated local plastic deformation.

The essential influence in fatigue damage is that of the local increase of stress and its irregular distribution around the most charged point, so call stress gradient.

Accordingly, fatigue of the whole construction is the defect in consistency of the construction due to the pulsing loading at lower level of the nominal stress than the level of the static strength limit, whereas the local material damage grows gradually with developing alternating plastic deformation until the fatigue crack occurs. The crack then keeps growing until the construction finally fractures.

9.2 – Definition of Alternating Load

Most of the machine parts and structures in operation are loaded by time-variable forces with repeated effects. A general case with variously large amplitudes of oscillation and of various nature of their time variability is called an alternating load.

If the force variability is always settled to the same highest and lowest limits at the same frequency of changes, it is called ***cyclic loading***, which is the ***special case of the alternating load***.

Considering that the alternating load can be superposed from a row of simple cyclic loads of various amplitudes and frequencies, it is suitable, in order to understand the changes caused by the variable effects in bodies, to observe the stabilized loading cycles and to study the law of the damage cumulation caused by superposition of the individual cyclic loads.

If the forces cause the stresses in due proportion, it is suitable to deal with the time behaviour of the stress. Analogically it is possible to deal with the time behaviour of the relative deformation.

A closed stress change that takes a continuous series of values is called a ***stress cycle*** (see fig. 9.2). The main characteristics of the stress cycle are:

a) Cycle period, i.e. time in which a closed single stress change occurs.

b) The largest s_h and the lowest s_d cycle stress i.e. limit values of stress within one cycle.

c) Mean cycle stress s_m

$$s_m = \frac{1}{2}(s_h + s_d),$$

d) Cycle stress amplitude, i.e. one half of an algebraic difference of the limit values of the cycle stress

$$s_a = \frac{1}{2}(s_h - s_d),$$

e) Stress range $2\,s_a$

f) Asymmetry coefficient of the stress cycle $R = \frac{s_d}{s_h}$.

This coefficient has the following values: for the alternating symmetrical stress $R=-1$, for the alternating unsymmetrical stress $-1<R<0$, for repeated stress $R=0$ and for pulsating stress $0<R<1$. We can imagine the above-stated stresses as the stresses consisting of two parts:

a) constant stress equal to the mean cycle stress s_m,

b) alternating (cyclic) stress of the amplitude s_a value.

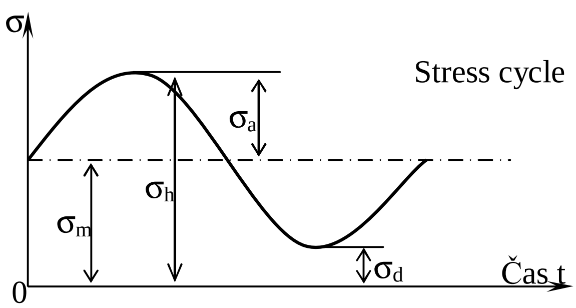

Fig. 9.2

*9.3 – **Wöhler's Curve***

The oldest background for evaluation of the material fatigue strength is the so called ***Wöhler's curve***. Wőhler was determining this curve more than one hundred years ago for bending during rotation on the cylindrical bars of the circular cross-section as the background for evaluation of strength of axletrees of railway carriages. It is necessary to stress that it is the curve of the cyclic strength, i.e. the curve representing one hundred percent fracture damage of a test bar. Thus it contains all the stages of creation and evolution of the fatigue cracks up to the final fracture, without distinguishing these stages. This curve has a large practical significance up to now as it is acquired in the simplest way, and for a long time it had been used as the basis for calculations, so it became common in technical practice.

The sample life, consisting of a stage of creation and development of the fatigue crack at cyclic loading, is, naturally, charged by a large dispersion. It shows that the Wöhler's curve cannot provide a reliable idea about the fatigue strength of smooth cylindrical bodies unless it is also complemented by the quantities expressing scatter of the test results. Statistic treatment of the fatigue tests is the essential condition of a sufficiently objective evaluation of the fatigue properties of material.

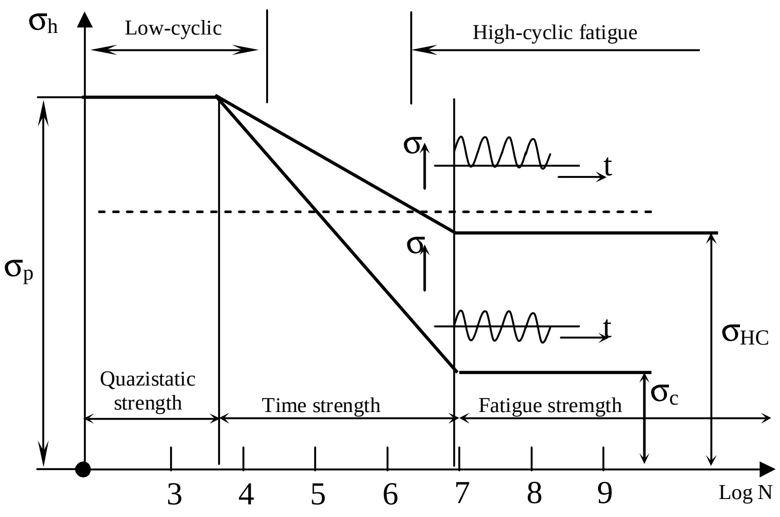

Fig. 9.3

The Wöhler's curve of smooth samples of technical metal materials has a typical character in the coordinates *s - log N* (see Fig. 9.3). The diagram can be divided into three typical sections. In the first part is situated the quasi-static strength (usually up to10^3 cycles). In the second part of the diagram there is a linear fall of the limiting strength with log N and thus the strength of the sample depends on the number of cycles. That is why this part is called ***area of the time strength***. Finally, in the third part of the diagram, usually in metals at normal temperature for the number of cycles above 10^7, the so called basic ***fatigue strength*** is reached, designated in the standards by the symbol s_c ***for alternating cyclic stress*** (symmetrical cycle) and by the symbol s_{HC} ***for repeated cyclic stress***. At $N>10^7$ the sample strength does not practically depend on the number of loading cycles.

In the area of the time strength the Wöhler's curve can be expressed by the power function

$$s_N{}^m N = s_C{}^m N_C$$

where N_C is the basic number of cycle for the fatigue strength s_C, equal usually 10^6 to 10^7 for steels (the basement of the fatigues tests).

9.4 – Safety Factor in the Area of the Time Strength

Equation (9.1) appears as a very suitable one for further mathematical treatment and shows sufficiently reliable results in the area of long time fatigue. For practical needs the exponent ***n*** is determined by means of two experimentally found points A_1, A_2 at the Wöhler's curve (see Fig. 9.4). Then the *n* is calculated from the relation:

$$n = \frac{\log N_2 - \log N_1}{\log s_{N1} - \log s_{N2}} = \frac{\log(N_2 / N_1)}{\log(s_{N1} / s_{N2})} \tag{9.2}$$

where s_{N1} and s_{N2} are stresses of alternating symmetrical cycles, which destroy material after N_1 and N_2 loading cycles. It is obvious from the Fig. 9.4 that the points A_1 *(log* N_1*; log* s_{N1}*)* and A_2 *(log* N_2*; log* s_{N2}*)* are situated on a straight line. As the value of the exponent ***n*** varies according to whether the same Wöhler's curve is expressed according to s_h or s_a, let us, hereafter, consistently consider $s_N = s_a$. If the part is subjected to n_1 cycles of symmetrical alternating stress with the amplitude s_{N1}, we can illustrate this state by the point M_1 *(*n_1*,* s_{N1}*)*.

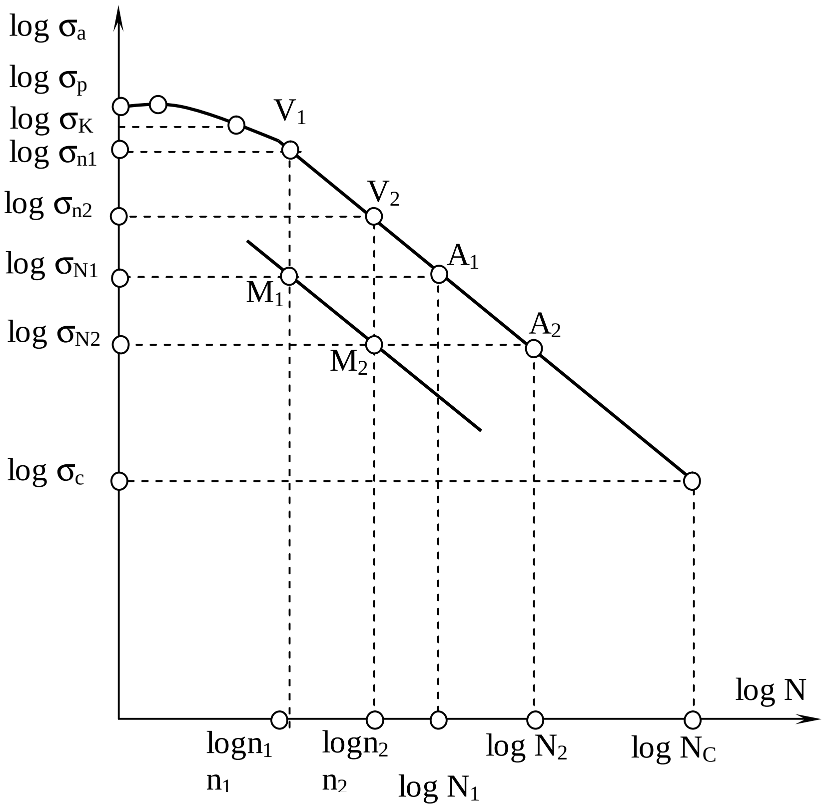

Fig. 9.4

If we draw parallel lines with the axes log *N* and log s_a through the point M_1 as far as the intersection point with the fatigue curve, we determine the <u>safety coefficient according to the stress</u>

from the relation $k_s = \frac{s_{n1}}{s_{N1}}$, where s_{n1} is time fatigue limit, corresponding to n_1 cycles of loading until the failure. ***The safety coefficient according to the number of cycles*** is given by the expression $k_N = \frac{N_1}{n_1}$, where N_1 is the number of alternating symmetrical cycles with the amplitude s_{n1}, at which the material fails.

As both points A_1 and V_1 lie on a fatigue curve expressed by the equation (9.1), we shall get

$$s_{N_1}^m N_1 = s_{n_1}^m n_1,$$

thus

$$\left(\frac{s_{n_1}}{s_{N_1}}\right)^m = \frac{N_1}{n_1},$$

hence it follows

$$k_s^m = k_N \quad . \tag{9.5}$$

The relation (9.5) shows, that there is a clear dependence between the safety coefficient according to the stress and the safety coefficient according to the cycle, the form of which is expressed by the relation (9.5), considering validity of the equation (9.1). If the real material limit of fatigue is σ_C, then it can be stated that the safety coefficient according to the fatigue limit is

$$k_C = \frac{s_c}{s_{N1}} \tag{9.6}$$

If $k_C<1$ (as, for example, for the point M_1 in the Fig. 9.4), then the failure occurs at the number N_1 of the cycles determined from the relation

$$N_1 = N_C\left(\frac{s_C}{s_{N1}}\right)^m$$

If $k_C>1$, the failure cannot occur at any number of cycles.

It is necessary to point out that the test results according to Wöhler show a considerable scatter. Therefore it is necessary to indicate a probability (reliability) of the values for each Wöhler's curve. Usually an average Wöhler's curve is used corresponding to the failure probability p =0,5.