6th Lecture

Design and Technological Stress Raisers

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6.1 – Introduction

Design stress raisers (Fig. 6.1) arise from the change of a forces-flow in the bodies because of notches, transitions and wall thickness changes.

The stress concentration causes also macro-defects of material such as cracks, laps, weld defects, shrinkage cavities and holes in casts etc.. These stress raisers are called **technological raisers**.

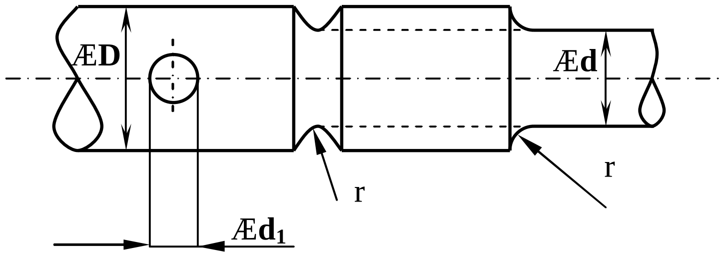

Fig. 6.1

6.2 – Influence of Local Design Stress Raisers

a. **Notches locally increase stress**, which can be expressed by a relative increase of stress in comparison with the level of nominal stress $\sigma_n = \sigma$, which would act in the bodies of the same type and identically loaded, but without a notch. This increase is expressed by a dimensionless stress concentration factor (sometimes called a shape factor):

$$a_s = a = \frac{s_{\max}}{s_n} > 1 .$$

b. **Local deformation** is increased in comparison with the nominal value of strain:

$$a_e = \frac{e_{\max}}{e_n} > 1 .$$

In the elastic area, where the proportionality $\sigma \sim \varepsilon$ is valid, there is:

$$a_s = a_e = a_{elteor} .$$

In the plastic area the strain concentration grows and the stress concentration falls:

$$a_e > a_s .$$

This fact depends on a non-linear relation between stress and strain. A re-distribution of stress occurs, which gradually balances.

c. **Notches cause change of stress in their neighbourhood** (triaxial state of stress occurs in rotary bars and voluminous bodies, biaxial state of stress in thin flat bars). However, the main stresses in the cross section under the notch are unevenly distributed (maxima correspond to the magnitudes of the corresponding stress concentrations α_1, α_2, α_3).

d. **<u>Stress concentration factor</u>** α depends on a:

- Curve radius of the notch root ρ,
- Notch depth t_v and dimension of the body in the main cross-section,
- Nature of stress (type of the forces-flow through the body)

<u>The sharper and deeper the notch is and the larger the body is, the higher is</u> a. In a number of cases the stress concentration is maximal at tensile loading, lower at bending and minimal at torsion loading.

The stress magnitude falls from the notch root. We talk about the stress gradient ***g.***

The largest gradient occurs at elastic state of stress in the notch root. In elasto-plastic state of stress the largest stress is at the interface of the plastic zone under the notch and elastic environment. At a larger distance from the notch root the stress falls under the level of the nominal value, because the total force or moment are determined by external loading and the stress must give internal forces being in balance with them . We then talk about release effect.

6.3 – Influence of a Circular Hole on the Stress Distribution in a Tensile Loaded Long Thin Band

The aim of this 6th Lecture is to determine the stress concentration factor in a long thin band with a central hole (see Fig. 6.2) that is tensile loaded.

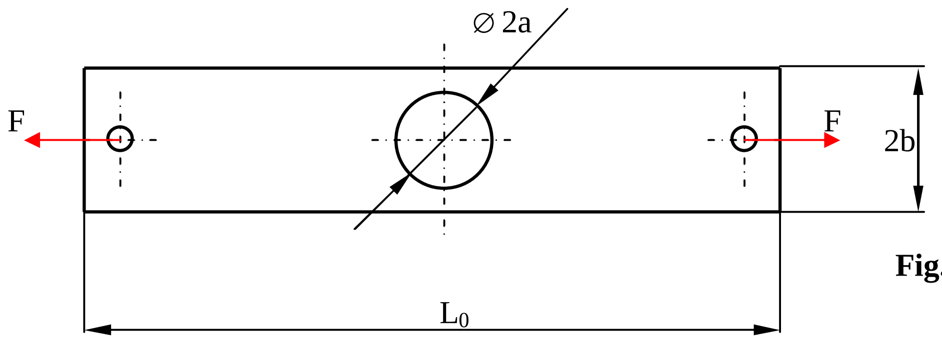

Fig. 6.2

The problem will be treated analytically, experimentally and numerically by means of the finite element method (FEM – model of the part).

A. Analytical Solution

In accessible technical literature, a solution of the problem depicted in the fig. 6.3 can be found. Stress in the neighbourhood of a circular hole of a radius ***<u>a</u>*** can be expressed in the following form:

$$s_r = \frac{s_0}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{s_0}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right) \cdot \cos 2q$$

$$\sigma_\theta = \frac{\sigma_0}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{\sigma_0}{2}\left(1+3\frac{a^4}{r^4}\right)\cdot\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma_0}{2}\left(1-3\frac{a^4}{r^4}+2\frac{a^2}{r^2}\right)\cdot\sin 2\theta$$

Significance of individual quantities given in the previous equations is obvious from the fig. 6.3. From the fig. 6.4 is then obvious loading of an element extracted from the area for $r \in \langle a, b\rangle$.

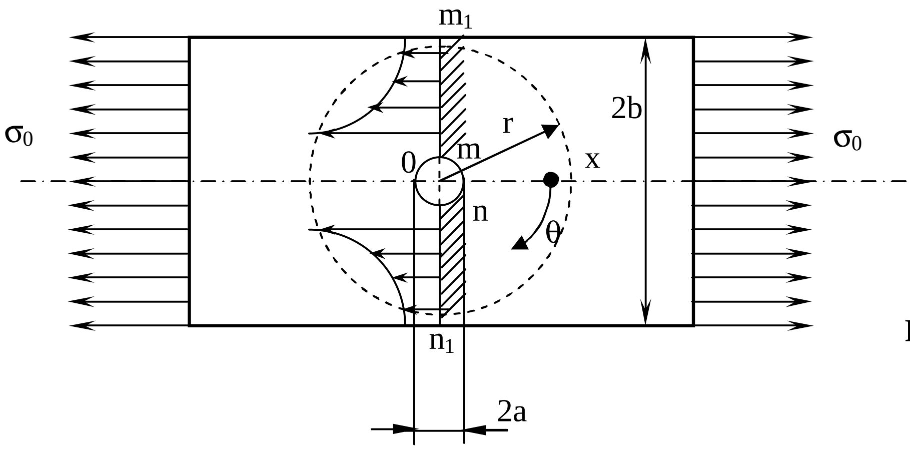

Fig. 6.3

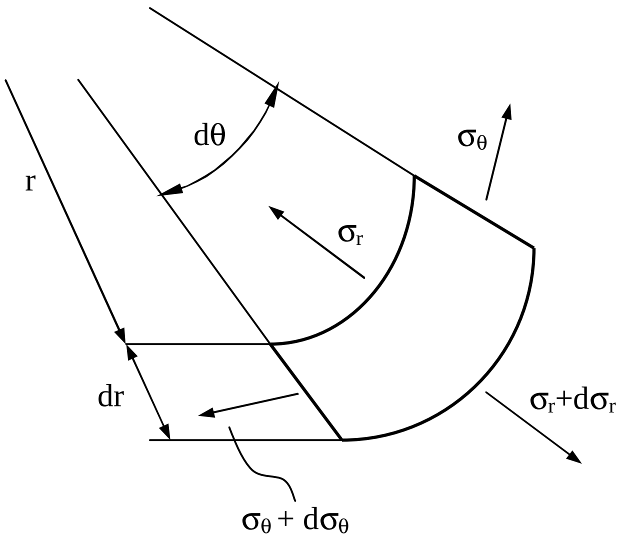

Fig. 6.4

On the hole surface, i.e. for $r = a$ we shall get:

$$\sigma_r = \tau_{r\theta} = 0 \quad ; \quad \sigma_\theta = \sigma_0 - 2\sigma_0 \cos 2\theta$$

It is obvious from the previous relation that the maximal magnitude of σ_θ is reached for the angle $\theta = \pi/2$ *or* $3\pi/2$. I those point is $\sigma_\theta = 3 \cdot \sigma_0$.

The stresses $\tau_{r\theta}$ *and* σ_θ in the cross-sections m_1, n_1, i.e. for $\theta = \pi/2$, are given by the relations:

$$t_{rq} = 0; \qquad s_q = \frac{s_0}{2}\left(2 + \frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)$$

We shall get the maximum again for $r = a$. i.e.

$$\max s_q = 3s_0 .$$

Further it is valid for the full cross-section:

$$s_0 = \frac{F}{2tb} .$$

The nominal stress in the reduced cross-section

$$s_{nom} = \frac{F}{2t(b-a)} .$$

The shape factor is

$$K = \frac{s_{\max}}{s_{nom}} = 3 \cdot \frac{\cancel{F}}{\cancel{2t}b} \cdot \frac{\cancel{2t}(b-a)}{\cancel{F}} = 3\left(1 - \frac{a}{b}\right) .$$

The above-stated expression for the shape factor **„*K*“** of the investigated problem is correct if the band width is considerably larger than the hole diameter. In case $b\ {}^{3}\ 4a$, the error in estimation of s_{max} of the given problem does not exceed 6%.

B. Experimental Solution

Experimental solution of the given problem will be the subject of the Laboratory tutorial.

C. Numerical Solution by Means of the Finite Element Method

Solution of the given problem will be the subject of the Laboratory tutorial.