

3<sup>rd</sup> Lecture

# **Strain Gauges for Static Measurement of Forces**

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### 3.1 – Introduction

By means of strain gauges it is possible to measure physical quantities that show themselves by material deformations. The strained elastic body must be designed in such a way that it is optimally sensitive for the required range of the measured quantity and sufficiently rigid at the same time. The sensor rigidity is usually required in order not to influence the measured quantity by the sensor deformation. The optimum design of the sensor assumes realization of the calculation of deformation of the measuring element, on which the strain gauges will be mounted. The computation can be realized by the methods of technical elasticity or by means of the Finite Element Method. Application of the FEM is suitable particularly in more complicated shapes of the measuring elements. The measuring elements, on which the strain gauges are installed, can by loaded by tension (pressure), bending or torsion. These ways of loading sensors will be dealt with in the following chapter. Loading of the prismatic bars by uniaxial tension or pressure and bending of long thin beams was subject of the 1<sup>st</sup> and 2<sup>nd</sup> Lecture.

The measurement precision is given by the applied type of the strain gauge (wire or foil, semiconductor), the measuring elastic element, the way of the force introduction and the calibration precision. Dispersion of the modulus of elasticity and dispersion of the k-factors of the installed strain gauges is eliminated by calibration. For measuring in a large temperature range it is necessary to respect the changes of the k-factor and of the modulus of elasticity of the elastic measuring element vs. temperature changes. This requirement can be fulfilled either by calibration at various temperatures or by a suitable temperature compensation.

#### 3.2 – Sensors of Uniaxial Tension and Pressure.

For measuring of large forces it is possible to use prismatic measuring elements loaded by tension or pressure. Parasite influence of bending or temperature can be eliminated by a suitable connection of the installed strain gauges into the measuring bridge. In Fig. 3.1 there is a connection of the measuring bridge with strain gauges installed on the prismatic bar loaded by tension. For simplicity, the illustrated bar is of the rectangle cross-section of the surface S=b.h. However, usually bars of cylindrical shape, either full or in the shape of an annulus, are used as the measuring elements. In case of the pressure loading it is recommended to have the ratio of the cylindrical measuring element length and its diameter approximately 3/2. The full bridge connection of two and two strain gauges at the opposite bar surfaces, given in the abovementioned picture, allows both temperature compensation and compensation of the parasite bending. The relations valid for the given type of the strain gauge for tension or pressure are as follows:

#### - Deformation

$$\boldsymbol{e}_1 = \boldsymbol{e}_2 = \frac{F}{E \cdot S} = \frac{F}{E \cdot b \cdot t} \quad ,$$

(3.1)

$$\boldsymbol{e}_3 = \boldsymbol{e}_4 = -\boldsymbol{u} \cdot \boldsymbol{e}_1 \quad , \tag{3.2}$$

- Displacement in the direction of the force F

$$u_F = \frac{F \cdot l_0}{E \cdot S} \quad , \tag{3.3}$$

- Change of the bridge balance



$$\frac{\Delta U_o}{U_i} = \frac{k_d \cdot e_1(1+u)}{2+k_d \cdot e_1(1-u)} , \qquad (3.4)$$

In the previous relations there are:

- $-\epsilon_i$ : (i=1,2,3,4) relative deformation at the point of installation of the i-th strain-gauge,
- E : modulus of elasticity of the measuring element material,
- $\upsilon$  : Poisson's coefficient of the measuring element material,
- k<sub>d</sub>: deformation coefficient of the used strain-gauges.

The equation (3.4) is derived from the equations given in the 2<sup>nd</sup> Lecture. Within the derivation it is supposed that the installed strain-gauges have the same idle resistance R. The mentioned sensor is compensated both thermally and against the parasite bending.





Fig. 3.1



## 3.3 – Sensors for Measuring the Forces by Means of Bending Stress

For measuring of rather small forces it is suitable to use measuring elements subjected to bending. In the Fig. 3.2 and 3.3 there is a sensor with a measuring element in the shape of a prismatic fixed beam. The relations valid for the depicted sensors are the following:

The half-bridge (Fig. 3.2):

- Deformation

$$\boldsymbol{e}_1 = \frac{\boldsymbol{6}\boldsymbol{F}\cdot\boldsymbol{l}}{\boldsymbol{E}\cdot\boldsymbol{b}\cdot\boldsymbol{h}^2} \quad , \qquad \boldsymbol{e}_2 = \boldsymbol{e}_1,$$

- Change of the bridge balance

$$\frac{\Delta U_o}{U_i} = k_d \cdot \frac{\boldsymbol{e}_1}{2} \quad .$$

The whole bridge (Fig. 3.3)

- Deformation  $e_1 = e_3 = \frac{6F \cdot l}{E \cdot b \cdot h^2}$ ,  $e_2 = e_4 = -e_1$ ,

- Change of the bridge balance

$$\frac{\Delta U_o}{U_i} = k_d \cdot \boldsymbol{e}_1 \quad .$$

For both connections it is valid:

- Displacement in the direction of the force F

$$u_F = \frac{4F \cdot l^3}{F \cdot h \cdot h^2}$$

In both cases the temperature influences are compensated and parasite tension and pressure are eliminated.



Fig. 3.2

Fig. 3.3