2nd Lecture

Measurement of Small Deformations by Means of Resistance Strain Gauges

Contents:

2.1 – Introduction

The resistance strain gauges are sensors for detection of relative deformations based on measuring of an intermediate quantity. They take advantage of the properties that the electric resistance changes when the conductor shows a deformation. The fact that some metal strings change their resistance owing to their deformation was published for the first time in 1856 by a Scottish mathematician and physicist William Thomson, lord Kelvin (1824-1907). This phenomenon can be quantified by means of the ratio of relative resistance variation and relative variation of length of the deformed sensor, i.e.

$$G = \frac{\frac{dR}{R}}{\frac{dL}{L}} = \frac{\frac{dR}{R}}{e_L} \quad ,$$

where R is initial resistance, L is initial length and e_L is relative deformation. The quantity G is often called a ,,gauge – factor“, or a strain coefficient k_d.

2.2 – Metallic Resistance Strain Gauges

Relation between the length variation and the conductor resistance.

The conductor resistance is a function of three variables:

$$R = r \cdot \frac{L}{A}$$

where R is ohmic resistance of the conductor, r is specific resistance of the conductor material, L is the conductor length and A is the conductor cross-sectional area. Small changes of the conductor resistance are caused by the change of all three quantities, as the conductor deforms, i.e.

$$dR = \frac{r}{A} \cdot dL - \frac{r \cdot L}{A^2} dA + \frac{L}{A} \cdot dr$$

By a simple modification of the equation (2.3) we shall get a relation for relative change of the deformed conductor in the following shape:

$$\frac{dR}{R} = \left[\frac{dL}{L} - \frac{dA}{A}\right] + \frac{dr}{r}$$

For simplification let us consider a circular cross section of the conductor, then for the change of the cross-sectional area of the conductor we can write:

$$dA = 2pr \cdot dr$$

where r is an initial radius of the conductor cross-section. By means of the above-given equations we can write:

$$\frac{dR}{R} = [e_L - 2e_r] + \frac{dr}{r}$$

where $e_L = \frac{dL}{L}$ is longitudinal and $e_r = \frac{dr}{r}$ is transverse deformation of the conductor.

The relation between ε_L and ε_r is given by the Poisson's number μ, i.e.

$$e_r = -m \cdot e_L$$

After the conversion we shall get: $$\frac{dR}{R} = \left(1 + 2m + \frac{1}{e_L} \cdot \frac{dr}{r}\right) \cdot e_L$$

Comparing the equation (2.8) with the equation (2.1) we shall get the following expression for the strain coefficient k_d, i.e.

$$k_d = 1 + 2m + \frac{1}{e_L} \cdot \frac{dr}{r}$$

It is obvious that the magnitude of the coefficient k_d depends on material (Poisson's number m, resistance r), but also on immediate magnitude of the longitudinal deformation e_L of the conductor. The dependence of the resistance variation on deformation is not necessarily linear; and really, considering larger areas of deformation, k_d is approximately constant only in some metals and alloys.

2.3 – Calibration of Strain Coefficient by Simple Bending

Considering the complicated nature of the strain coefficient of the strain-gauge, it is suitable to determine this coefficient experimentally on a ready strain-gauge. In case the gauge body is in the shape of a long thin flat strip, a testing device based on the principle of a beam loaded by simple bending is usually used. In the following part let us focus on bending of a straight beam of the constant cross section through the methods of technical elasticity.

Strain and stress of the filaments of a long straight beam at simple bending.

Each cross section of the bended beam transmits a bending moment M_0 and shearing force T (see Fig. 2.1). Let us investigate the momentum and shearing force behaviour by the method of virtual section.

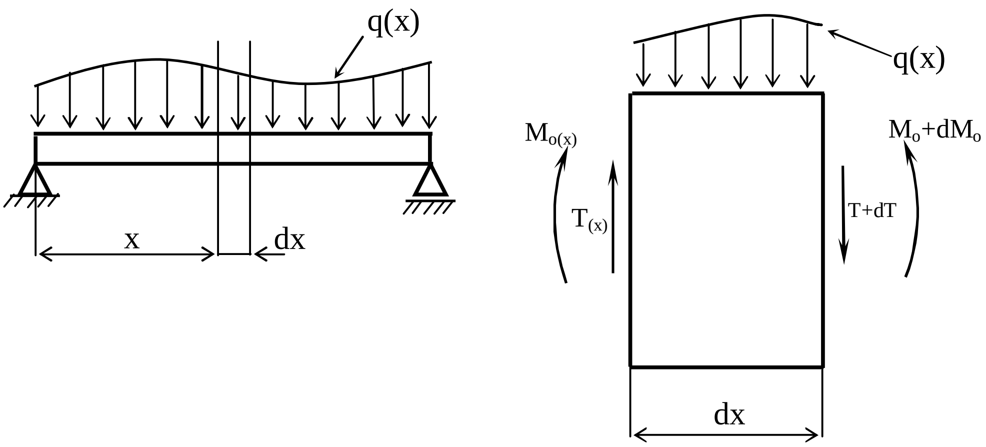

Fig.2.1

Let us remind that the behaviour of the bending moment and the shearing force are caused by external loading of the beam. By means of the balance equations of an extracted beam element (see Fig.2.1) differential relations between the bending moment M_0, shearing force T and external loading q(x) can be formulated. These relations are called Schwedler's sentences. It is valid:

$$\frac{dM_0(x)}{dx} = T(x) \quad \text{and} \quad \frac{dT(x)}{dx} = -q(x) \quad . \qquad \text{(2.1 and 2.2)}$$

At simple bending M_0=constant. It is obvious from the equation 2.1 that at simple bending the shearing force T(x) is identically equal to zero. For simple bending so called Bernoulli hypothesis is established in technical elasticity. According to this hypothesis the plane cross-sections perpendicular to the longitudinal axis of the beam before the deformation will remain plane after the deformation as well and will be perpendicular to the deformed longitudinal axis of the beam (see Fig. 2.2).

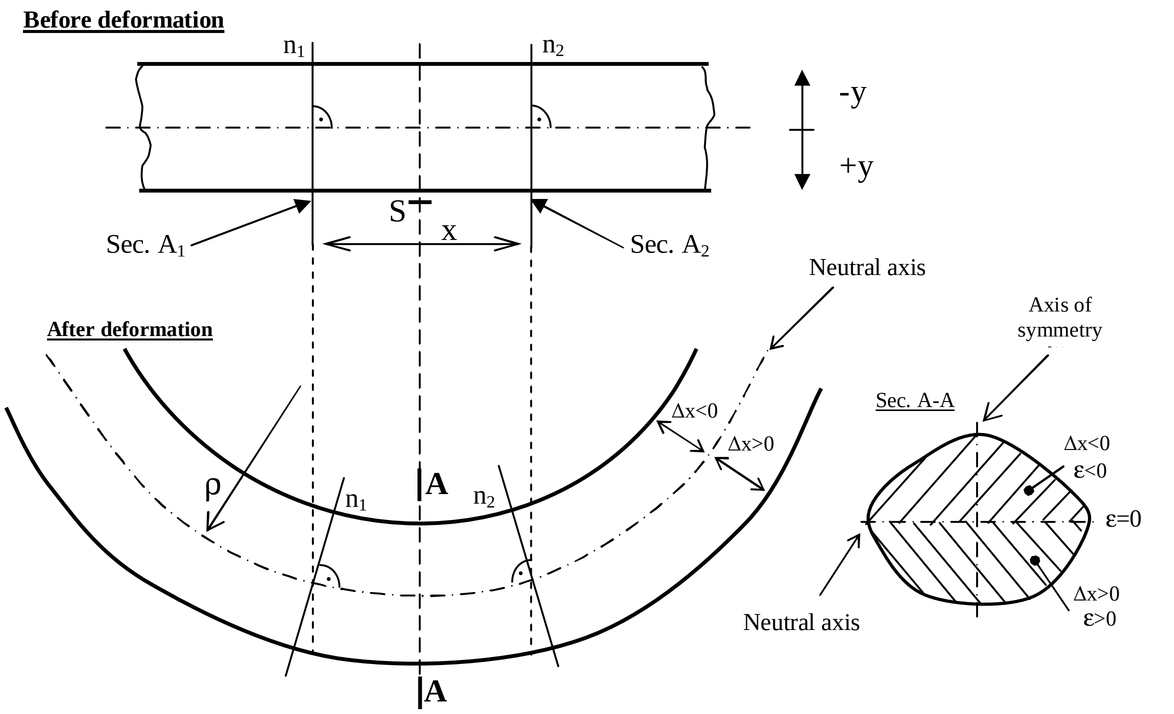

Fig. 2.2

If we draw virtual longitudinal filaments through the beam, then it is clear from the Fig.2.2., that part of these filaments are prolonged, part of them are shortened and part of them do not change their length. The filaments that do not change their length fill so called neutral area. The neutral area crosses every cross-section at neutral axis. Providing the beam does not transmit any axial force N it can be shown that the neutral axis during bending has to pass through the centre of gravity of the cross-section.

Let us denominate ρ the flexure radius of the neutral area. Let us denominate j the angle of displacement of the sections A_1 and A_2 after deformation. Then the length of the unstrained filament is:

$$L_0 = r \cdot j$$

The length of the strained filament is:

$$L = (r + y)j$$

Relative deformation of the filament at the distance *y* from the neutral area is:

$$e(y) = \frac{L - L_0}{L_0} = \frac{(r + y)\cdot j - r\,j}{r\,j} = \frac{y}{r} \qquad (2.3)$$

Simple bending is uniaxial state of stress for which the Hooke’s law is valid.

$$s = E \cdot e$$

i.e.

$$s(y) = E \cdot \frac{y}{r} \quad . \qquad (2.4)$$

The relation (2.3) for deformation and the relation (2.4) for stress show that the largest magnitudes are reached in the edge filaments i.e. in the filaments the most distant from the neutral axis. By means of the equation of the momentum balance to the neutral axis we shall find the relation between torsion ρ of the neutral plane and the bending moment at the given cross-section of the beam (see Fig.2.3).

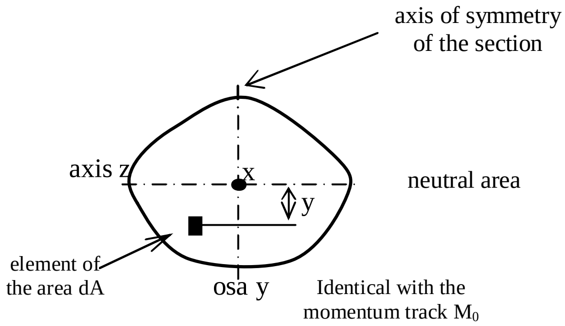

Fig. 2.3

Momentum condition

$$\int_A y \cdot s(y) \cdot dA - M_0 = 0\,, \qquad (2.5)$$

Substituting equation (2.4) for σ(y) in the equation (2.5) and after the conversion we shall get:

$$\frac{1}{r} = \frac{M_0}{E \cdot J_z} \quad , \qquad (2.6)$$

where

$$J_z = \int_A y^2 \cdot dA \qquad (2.7)$$

is quadratic moment of the cross-section to the neutral axis, identical with the axis *z*.

Substituting the relation (2.6)´ in the relation (2.3) for the beam filament deformation we shall find the relation between the moment M_0 of external forces and deformation of the filament within the distance *y* from the neutral axis. Thus it is true:

$$e(y) = \frac{M_0}{E \cdot J_z} \cdot y \qquad (2.8)$$

It is obvious from the relation stated above that the filament strain is given by:

- External loading (bending moment M_0)
- Material rigidity (Young's modulus of elasticity *E*)
- Geometrical shape of the cross-section (quadratic moment of the cross-section J_z)

Let us remark that the product (E J_z) is denominated as the flexural rigidity of the beam.

The relation between the stress in the filament and the external loading results from the substitution of the relation (2.6) into the relation (2.4), i.e.

$$s\,(y) = \frac{M_0}{J_z} \cdot y \qquad (2.9)$$

As it has already been written, the maximal stress is reached in the edge filaments, i.e. for $y=y_{max}$, thus:

$$s_{\max} = \frac{M_0}{J_z} \cdot y_{\max} \qquad (2.10)$$

or, if you like

$$s_{\max} = \frac{M_0}{W_0} \qquad (2.11)$$

where

$$W_0 = \frac{J_z}{y_{\max}}$$

The quantity W_0 is a characteristic of the shape of the cross-section only and is called a modulus of bending resistance of the cross-section.

The relations for J_z and W_0 for common shapes of the cross sections can be found in the Technical Data Sheets.

2.4 – Measurement of Small Resistance Variations of a Strain Gauge

2.4.1 Bridge connection

The most common method of measuring small resistance changes of strain gauges is the bridge connection, usually called Wheatstone bridge (Sir Charles Wheatstone 1802-1875).
The connection diagram is shown in the Fig. 2.4

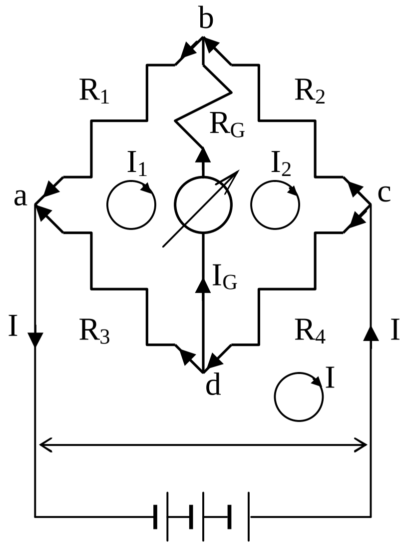

Fig. 2.4

The change of resistance of some of the resistors R_1 to R_4 results in the change of electric current I_G passing through the measurement diagonal (galvanometer). The magnitude of I_G can be determined By means of the Kirchhoff's Laws.

Let us recall the above-mentioned Kirchhoff's Laws:

I. Algebraic sum of the currents in any point (nod) equals to zero, i.e.

$$\sum_{1}^{h} I_K = 0 \ .$$

II. Sum of all applied voltages in the closed circuit equals to zero

$$\sum_{1}^{h} U_K = 0 \ .$$

For completeness' sake let us mention also the Ohm's Law: U = R·I .

Thus for the bridge given in the Fig. 3.4 it is true:

$$I_G = \frac{U_G}{D} \cdot (R_1 \cdot R_4 - R_2 \cdot R_3) \ , \tag{2.12}$$

where the determinant D in the denominator of the equation (3.12) is given

$$\begin{vmatrix} -R_2 & -(R_3+R_4) & R_3+R_4 \\ -R_G & R_1+R_3 & -R_3 \\ R_2+R_4+R_G & R_2+R_4 & -R_4 \end{vmatrix}$$

When the bridge is balanced, there is no current passing through the galvanometer, i.e. I_G=0.

The equation (3.12) results in the relation:

$$R_1 \cdot R_4 = R_2 \cdot R_3 \quad . \qquad 2.13$$

If the magnitude of resistors changes in the balanced bridge, for example R_1 changes by ΔR_1, the bridge unbalances. The magnitude of ΔR_1 can be determined in two ways.

a. Zero Method

This method uses balancing of the bridge by adding resistors in the other branches so that again I_G=0 was valid. Let us realise balancing by a suitable change of the resistance R_2.

Then
$$(R_1 + \Delta R_1) \cdot R_4 = (R_2 + \Delta R_2) \cdot R_3 \quad . \qquad (2.14)$$

Herefrom the measured change ΔR_1 is given by the relation

$$\Delta R_1 = \Delta R_2 \frac{R_3}{R_4} = konst. \Delta R_2 \quad . \qquad (2.15)$$

The Zero Method can be applied in static measurements, when there is time enough for the balancing. The advantage of this method is independence of the method accuracy on the variation of the voltage U_G.

b. Deviation Method

When applying this method, the bridge is not being balanced, but the magnitude of the current I_G is measured directly. If the bridge is balanced and the resistance R_1 changes by ΔR_1, then the change of current in the measuring branch is

$$\Delta I_G = \frac{U_G}{D} \cdot R_4 \cdot \Delta R_1 \quad . \qquad (2.16)$$

Considering that the change ΔR_1 is very small when compared with R_1, the influence of ΔR_1 on the magnitude of the determinant D can be neglected. Then D'=D and hence

$$\Delta I_G = \frac{U_G}{D} \cdot R_1 \cdot \Delta R_1 = const.\, \Delta R_1 \quad . \qquad (2.17)$$

2.4.2 Bridge fed by a.c.-voltage

At bridges fed by AC voltage it is necessary to consider an impedance resistor instead of the ohmic one, i.e.

$$Z = R + jX \ ,$$

where X is reactance

The balance conditions are $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ a $\frac{X_1}{X_2} = \frac{X_3}{X_4}$.

For the bridge balance it is necessary not only to balance the ohmic resistors but also reactance, usually those of the capacities. If resistance change ΔR_1 occurs at the ohmic and capacity balanced bridge, the change of the current I_G is proportional to this change.

2.4.3 Bridge as a voltage divider

Let two resistors R_1 and R_4 be put on series and connected to the constant voltage supply U_G (see Fig. 2.5a)

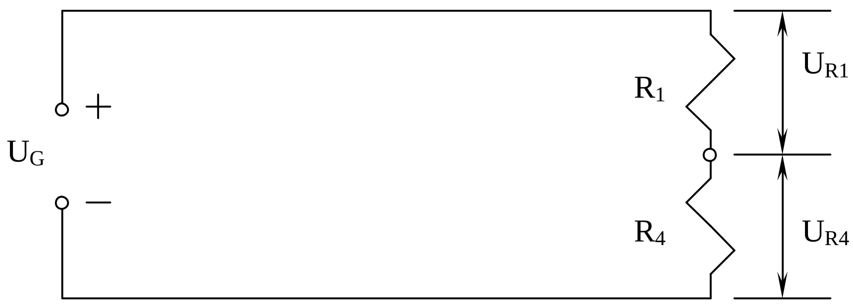

Fig. 2.5a

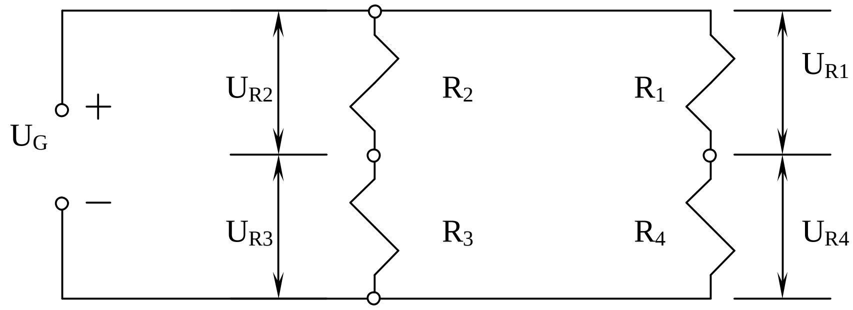

Fig. 2.5b

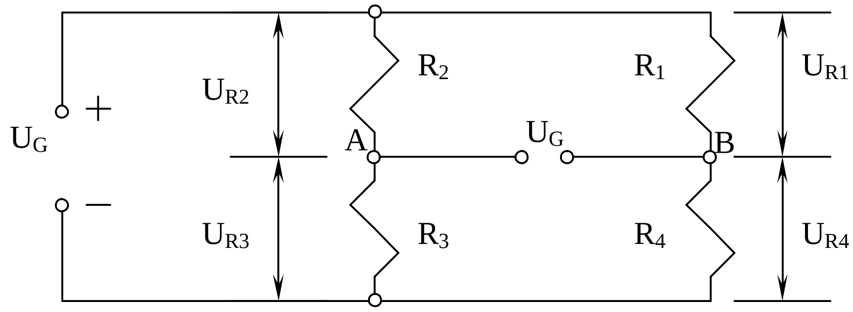

Fig. 2.5c

The following relations are valid

$$\frac{U_{R1}}{U_{R2}} = \frac{R_1}{R_4} \quad . \tag{2.18}$$

$$U_{R1} = U_B - U_{R4} \quad . \tag{2.19}$$

Then from the previous relations

$$U_{R1} = U_B \frac{R_1}{R_1 + R_4} \quad , \tag{2.20}$$

$$U_{R4} = U_B \frac{R_4}{R_1 + R_4} \quad . \tag{2.21}$$

Voltage Bridge

The voltage bridge can be considered as parallel connection of two dividers of voltage (see Fig. 2.5b). It is obvious that for the drop of potential between the points A and B it is valid:

$$U_G = U_{R1} - U_{R2} \quad , \tag{2.22a}$$

$$U_G = U_{R3} - U_{R4} \quad . \tag{2.22b}$$

For the parallel divider and the resistors R_2 and R_3 we can write:

$$U_B = U_{R2} + U_{R3} \quad . \tag{2.23}$$

$$U_{R2} = U_B \cdot \frac{R_2}{R_2 + R_3} \quad , \tag{2.24}$$

where

$$U_{R3} = U_B \cdot \frac{R_3}{R_2 + R_3} \quad . \tag{2.25}$$

Substituting the relations (2.20) and (2.24) into the equation (2.22a) we shall get an expression for the output voltage U_G in the following shape:

$$U_G = U_B \cdot \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \tag{2.26}$$

It is clear from the equation 2.26, that the bridge is balanced, i.e. $U_G=0$ on condition that

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \quad . \tag{2.27}$$

If a resistance change by ΔR_1 occurs at the sensor R_1, then it can be shown through the derivation of the equation (2.26), that the variation of the output voltage U_G is given by the relation:

$$\Delta U_G = U_B \cdot \frac{\Delta R_1}{R_1} \cdot \frac{R_1 \cdot R_4}{(R_1 + R_4)^2} \quad . \tag{2.28}$$

When measuring with the bridge connection, usually the resistors are selected to be $R_1 = R_2 = R_3 = R_4 = R$. In that case when the resistance of one bridge changes by ΔR, the output voltage at the diagonal of the bridge will be as follows:

$$\Delta U_G = \frac{\Delta R}{4R} \cdot U_B \quad . \tag{2.29}$$

If the resistance change occurs simultaneously in two branches, the output signal from the bridge is given by the remainder of the voltage changes in case that the resistance change occurred in the adjacent branches. In case the change occurred in the opposite branches, the output signal is given by the addition of the voltage changes.

If the resistances in the adjacent branches are changed in the opposite direction, the resulting signal is given by their addition, i.e. with the resistance change of the same magnitude at the adjacent resistors the bridge remains in balance. With the resistance change of all four branches the resulting signal is four times as big, if the resistance change is of the same magnitude and opposite signs in the adjacent branches. Validity of the above-mentioned statements can be verified through the relation

$$\Delta U_G = U_B \cdot \left[\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4 + \Delta R_4} - \frac{R_1}{R_1 + R_4} - \frac{R_2 + \Delta R_2}{R_2 + \Delta R_2 + R_3 + \Delta R_3} - \frac{R_2}{R_2 + R_3} \right] \quad . \tag{2.30}$$

Temperature Compensation

This compensation is performed by means of the compensation strain gauge placed on the same material as the measuring strain gauge but not submitted to the strain (see Fig. 2.6). If there is a temperature change during the measurement, it causes the same resistance change both at the measuring gauge and the strain gauge. If both these gauges are connected in the bridge in one parallel branch, the resistance change is eliminated by the bridge itself.

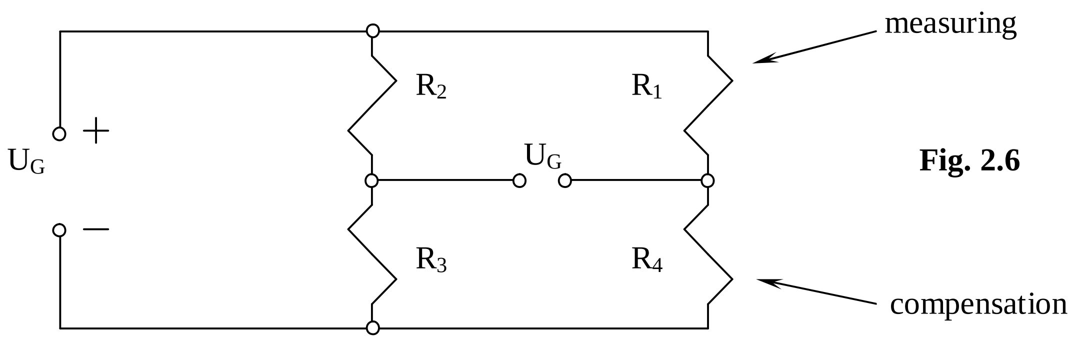

Fig. 2.6

The above given statement can again be proved through the relation (2.30).